Sun Sensor Navigation for Planetary Rovers: Theory and Field Testing

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Abstract

In this paper, we present an experimental study of sun sensing as a rover navigational aid. Algorithms are outlined to determine rover heading in an absolute reference frame. The sensor suite consists of a sun sensor, inclinometer, and clock (as well as ephemeris data). We describe a technique to determine ground-truth orientation in the field (without using a compass) and present a large number of experimental results (both in Toronto and on Devon Island) showing our ability to determine absolute rover heading to within a few degrees.

Keywords: sun sensor, planetary rover, celestial navigation, localization

I. INTRODUCTION

Navigation for planetary rovers relies on integrated local measurements, most often from combinations of visual odometry, wheel tachometer readings, and inertial navigation sensors. These relative navigation schemes are subject to unbounded growth of errors over time/distance. Imagery from orbiting satellites can be used to provide global position corrections, but these updates are generally infrequent and introduce substantial latency if the imagery must be processed on the ground. Moreover, they give little information about the rover's heading. Absolute heading information can be calculated from digital compass readings, but the most likely targets for rover exploration (i.e. the moon and Mars) do not have useful magnetic fields. Measuring the sun direction with a high-accuracy sun sensor is an effective means of obtaining absolute orientation, and thereby reducing the drift in navigation accuracy during autonomous navigation.

This paper examines the problem of rover navigation with the aid of sun sensors. We outline the mathematical basis of our headingdetermination and evaluate the algorithms on data collected through a number of field trials conducted in the summer of 2008. Tests were conducted at the University of Toronto Institute for Aerospace Studies (UTIAS) and on Devon Island in the Canadian Arctic as part of the Haughton-Mars Project. The Devon Island terrain is a popular analog to the Martian surface and represents a good environment to evaluate rover

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navigation performance. Tests were conducted using a Sinclair Interplanetary SS-411 digital sun sensor and a suite of conventional rover sensors mounted on a mobile platform.

The paper is organized as follows. First, we review prior work on sun sensors for planetary rovers. Next, we present the theoretical development of our algorithms to determine both absolute rover heading and absolute global position. We then provide the details of our field testing campaign and experimental results. A conclusion follows.

II. PRIOR WORK

The inclusion of a sun sensor on future rover missions was one of the recommendations coming out of the 1997 Mars Pathfinder mission [2]. This recommendation was implemented on the 2004 Mars Exploration Rovers (MERs). Eisenman et al. [3] describe the method the MERs use to transform PanCam images into sun vectors. They do not describe the method of solving for orientation given the sun vectors. Ali et al. [4] also describe the uses of sun sensing on the MERs. The MERs use sun sensing in two different ways: (i) to update the heading of the rover and (ii) to update the full attitude of the rover (which is used to point the high gain antenna). The algorithms are not described, although they say that they use "QUEST" and cite Schuster and Oh [5]. Accuracy of the technique is not reported. We treat a similar problem in our Experiment 2 (Heading from Sun and Position) below. Maimone et al. [6] report that "As of January 2007, sun-locating commands have run more than 100 times on each rover"

Kuroda et al. [7] describe a lunar global/local localization method using sun vectors and Earth vectors. They perform simulations and find that, using sun and Earth vectors alone, they are able to estimate their global position on the moon to within 0.5 km. The authors develop a method of linking consecutive global localization readings using dead-reckoning. They claim that this method increases the accuracy of the estimation to within 150 m. The authors do not report any results of hardware experiments nor do they include a model of inclinometer error in their simulations.

Several JPL studies have also used rover-based sun sensing. Volpe [8] describes the field testing of the relative localization system on the Rocky 7 field rover. The sun sensor heading determination scheme limits the cross-track error to linear growth as the rover travels. Trebi-Ollennu et al. [9] describe the design and testing of a sun sensor on one of the FIDO rover platforms at JPL. They report errors in rover heading of a few degrees, which is comparable to our Experiments 1 and 2 below. Sigel and Wettergreen [10] perform some mathematical modeling and simulation for localization on the moon using a star tracker. They comment that the major source of error is the inclinometer.

Deans et al. [11] develop a sun sensor based on a fisheye-lens firewire camera and an inclinometer bundled together. They describe their frames of reference, the calibration procedure for the system, and plot inclinometer and image residuals after calibration. To find orientation, they use a weighted least-squares cost function, which they optimize using a Levenberg-Marquardt algorithm. They assume their position on the planetary body is known. These assumptions are similar to the ones we make in our Experiment 1 (Heading from Sun, Gravity, and Position). Deans et al. state, "The sensor orientation is estimated to within a fraction of a degree, but we lack a simple and independent method for measuring the ground truth orientation with similar precision." The issue of ground-truth heading (particularly in a remote field setting) is a challenge that we attempt to address in the current paper.

Pingyuan et al. [12] describe a method of attitude determination on the moon using sun, Earth and gravity vectors. They formulate the problem similarly to the current paper and also use the q-method to solve for attitude. They test their system on one simulated traverse and conclude that their method will estimate orientation. This is similar to our Experiment 1 (Heading from Sun, Gravity, and Position), except that in Pingyuan et al., it is assumed the starting global position (i.e., latitude/longitude) is known and propagated through an odometry relative position estimate as the rover moves. This allows the global position to be known each time heading is estimated during the traverse.

Cozman and Krotkov [13] also use the "circles of equal altitude" method to solve for the position of a sun sensor. The authors develop a sun sensor using a camera and inclinometer. Altitude values are corrected for parallax and refraction. They minimize a least-squares cost function using a grid of initial guesses, keeping the best (lowest error) solution. They perform two experiments that are based on only two sun observations each. They report the inclinometers as a major source of error.

Our paper seeks to extend these previous works in the following ways: (i) by providing extensive experimental results of determining heading using a sun sensor (i.e., our datasets were collected at two very different sites on Earth and contain thousands of sun vectors), (ii) by studying the sensitivity of heading error to sun elevation and rover position, (iii) by providing a method to self-calibrate the relative orientation between the sun sensor and inclinometer sensors, (iv) by providing a method to obtain ground-truth rover heading in the field (i.e., without a magnetic compass) and a means to predict its accuracy, and (v) by providing studies of the effect of sun sensor window length on the resulting heading estimate accuracy. The issue of determining global position using a sun sensor and inclinometer is beyond the scope of the paper.

III. MATHEMATICAL FRAMEWORK

This section provides the mathematical framework for the analysis used in this paper. We first define the key frames of reference used in our measurements, explain the calculation of the predicted vectors, and then solve for the rover orientation that best matches the observations. We also consider how sensor data can be combined in different ways to solve for the relevant set of unknowns. Each configuration effectively defines a separate *experiment* (which are described below).

A. Frames of Reference

Working with vector observations is an exercise in frame transformations. Relating sensor observations to ephemeris predictions relies on a clear understanding of the relevent reference frames. Six reference frames are of particular importance to this study (Table I). The Earth-Centred Inertial (ECI) and Earth Centred Fixed (ECF) frames follow standard definitions. Rover heading and motion is best understood relative to the local topocentric frame. This frame is defined with respect to the local horizontal. Strictly speaking, gravity is measured normal to the geoid, but since the *deflection of the vertical* is always less than 0.01°[14], we assume that gravity is also normal to the reference Earth ellipsoid. The remaining three frames are defined with respect to the sun sensor,

TABLE I
RELEVANT FRAMES OF REFERENCE

Frame	Notation	<i>x</i> -axis <i>y</i> -axis		<i>z</i> -axis
Earth Centred Inertial (ECI)	Ι	Vernal Equinox		North Pole
Earth Centred Fixed (ECF)	F	Prime Meridian		North Pole
Topocentric	T	Local East	(Local North)	Opposite Gravity
Sun Sensor	S		Alignment Pins	Outward Normal
Inclinometer	G	Away From Connector		Outward Normal
Camera	C	Horizontal Pixels	Vertical Pixels	(Optical Axis)



Fig. 1. Earth Centred Inertial, Earth Centred Fixed, and Topocentric reference frames. See Figure 9 for the Sun Sensor, Inclinometer, and Camera frames.

inclinometer, and camera housings, respectively. For this study, we consider F to be the primary external frame and S to be the primary rover frame. See Figures 1 and 9 for depictions of the references frames used throughout the paper.

Because common notations differ slightly, it is worth mentioning our convention. Vectors and matrices are expressed in boldface (e.g., \mathbf{x}_b). Subscripts denote the frame of reference for the vector. All of our vector transformations are rotations, captured in transformation matrices of the form \mathbf{C}_{ab} , read as: the rotation from b to a. Hence, the transformation of a vector is written

$$\mathbf{x}_a = \mathbf{C}_{ab} \mathbf{x}_b. \tag{1}$$

It is often convenient to express frame transformations as a series of principal axis rotations. Using the shorthand $c_{\theta} \equiv \cos \theta$, and $s_{\theta} \equiv \sin \theta$, our notational convention defines the three standard rotations:

$$\mathbf{R}_{x}\left(\theta\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\theta} & -s_{\theta} \\ 0 & s_{\theta} & c_{\theta} \end{bmatrix}, \quad \mathbf{R}_{y}\left(\theta\right) = \begin{bmatrix} c_{\theta} & 0 & s_{\theta} \\ 0 & 1 & 0 \\ -s_{\theta} & 0 & c_{\theta} \end{bmatrix}, \quad \mathbf{R}_{z}\left(\theta\right) = \begin{bmatrix} c_{\theta} & -s_{\theta} & 0 \\ s_{\theta} & c_{\theta} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(2)

The geometric interpretation of this convention is that if $C_{ab} = R_x(\theta)$, then b is obtained from a by a right-handed



Fig. 2. Frames associated with the sensor head. Clockwise from top-left: the topocentric frame, T, (x east, y north); the sun sensor frame, S; the inclinometer frame, G; and the camera frame, C.

rotation of θ about the x-axis. Given these definitions, we next derive expressions for converting between the frames of interest.

The transformation from ECF to ECI is a simple rotation:

$$\mathbf{C}_{IF} = \mathbf{R}_{z} \left(\Psi \right), \tag{3}$$

where Ψ is the Greenwich Apparent Sidereal Time (GAST). This study uses a 'low' accuracy algorithm [15] to calculate Ψ , which is accurate to about 1.3×10^{-4} degrees.

The transformation between T and F is a function of the rover longitude, λ , and latitude, ϕ . From the definitions of these frames we can show that

$$\mathbf{C}_{FT} = \mathbf{R}_{z} \left(\lambda \right) \mathbf{R}_{y} \left(\frac{\pi}{2} - \phi \right) \mathbf{R}_{z} \left(\frac{\pi}{2} \right).$$
(4)

Three further rotations in roll, γ , pitch, β , and heading, α , are used to calculate the transformation between the sun sensor and topocentric frames (the heading is measured counter clockwise from East):

$$\mathbf{C}_{TS} = \mathbf{R}_{z}\left(\alpha\right) \mathbf{R}_{y}\left(\beta\right) \mathbf{R}_{x}\left(\gamma\right) \tag{5}$$

The final frame transformations of note are between the sun sensor and the inclinometer and camera frames: C_{SG} , C_{SC} . This transformation captures the intended mounting geometry between the three sensors, as well as any misalignment between sensor axes. We do not assume a particular functional form for these matrices; they are obtained from calibration (see Section III-E for details of the calibration procedure).

Heading and position determination rests on our ability to relate measured vectors to predictions of what those vectors should be. The sun vector from the sun sensor can be related to the sun vector predicted from solar ephemeris. Likewise, gravity (as measured by our inclinometers) can be related to the expected gravity vector defined by our position.

1) Ephemeris Predictions: The expected sun vector in ECI, s_I , can be calculated from quasi-analytical models of solar ephemeris. Our sun sensor measurements are accurate to about 0.1° , so we can use a low accuracy prediction of the sun position. For this study we employ Meeus' solution [15], accurate to about 0.01° within a few centuries of the year 2000. These calculations rely on an accurate time reference. For convenience, our experiments use GPS timestamps but a real planetary rover mission would require an accurate on-board local clock and a method to calibrate it. Coarse clock synchronization (i.e., to within tens of milliseconds) over interplanetary distances are standard practice [16] and would be adequate for our purposes.

Sun vectors are rotated into ECF using C_{FI} , obtained from (3). This gives the 'predicted sun vector', s_F :

$$\mathbf{s}_F = \mathbf{C}_{FI} \mathbf{s}_I \tag{6}$$

If rover position is known, the predicted gravity vector, \mathbf{g}_F , follows from simple geometry:

$$\mathbf{g}_F = - \begin{bmatrix} c_\phi c_\lambda \\ c_\phi s_\lambda \\ s_\phi \end{bmatrix}$$
(7)

The sense of this vector is defined so that it points *down* along the local vertical. It is interesting to note that this expression holds regardless of whether ϕ is the geocentric or geodetic latitude.

2) Refraction Corrections: Incoming sunlight is refracted as it passes through the atmosphere causing an increase in the apparent elevation of the sun above the horizon. Refraction is irrelevant for airless bodies such as the moon, but must be accounted for during terrestrial trials. Although it is feasible to correct the *sensor's* sun vector readings, it is simpler for us to correct the ephemeris predictions, s_F , to reflect the sun's apparent position. In terms of the true, ε_{true} , and apparent elevation angles, ε_{app} , this takes the form

$$\varepsilon_{\rm app} = \varepsilon_{\rm true} + \Delta \varepsilon.$$
 (8)

The true elevation can be computed directly from the ECF sun and gravity vectors:

$$\varepsilon_{\mathrm{true}} = \mathrm{acos}\left(\mathbf{s}_{F}^{T}\mathbf{g}_{F}\right) - \frac{\pi}{2}$$

Saemundsson [17] provides the following formula for the elevation correction (in radians):

$$\Delta \varepsilon = \frac{2.97 \times 10^{-4}}{\tan\left(\varepsilon_{\rm true} + \frac{0.180}{\varepsilon_{\rm true} + 0.0892}\right)} \tag{9}$$



Fig. 3. Cutaway through the Earth showing geometry of refraction correction. Plane of diagram contains the sun, the sun sensor, and the center of the Earth.

The corrected sun vector, \mathbf{s}'_F , can be calculated from

$$\mathbf{s}_F' = \sin \varepsilon_{\rm app} \, \mathbf{h}_F - \cos \varepsilon_{\rm app} \, \mathbf{g}_F,\tag{10}$$

where the local horizontal vector towards the sun, \mathbf{h}_F , is

$$\mathbf{h}_F = \frac{-\mathbf{g}_F^{\times} \mathbf{g}_F^{\times} \mathbf{s}_F}{\cos \varepsilon_{\text{true}}}.$$
(11)

The geometry of this refraction correction is shown in Figure 2.

3) Onboard Measurements: The sun sensor itself provides s_S , the 'measured sun vector'. The 'measured gravity vector', g_S , can be calculated from the inclinometer readings. This instrument provides local pitch, β_G , and roll, γ_G , measurements such that

$$\mathbf{g}_{G} = \begin{bmatrix} s_{\beta_{G}} \\ -c_{\beta_{G}} s_{\gamma_{G}} \\ -c_{\beta_{G}} c_{\gamma_{G}} \end{bmatrix}.$$
(12)

Transforming the gravity vector into the sun sensor frame gives:

$$\mathbf{g}_S = \mathbf{C}_{SG} \mathbf{g}_G,\tag{13}$$

where C_{SG} is a calibration rotation matrix from the inclinometer frame to the sun sensor frame; we will describe how to determine this below.

Figure 3 compares the solar ephemeris predictions with the sun sensor data from a nine-hour test on September 10, 2008. The plots illustrate the observed range of solar motion, and the errors in the predictions over the course of the test. Irregularities in the observations near 15:45 and 17:15 are caused by partial cloud cover and gaps near 19:00 are caused by heavier cloud cover. Slow changes in the error suggest that some systematic biases are still present. Possible sources of these biases include: (i) inclinometer and/or sun sensor temperature drift and (ii) miscalibration of the rotational transformations between the references frames attached to the sensors.



Fig. 4. Predicted and observed sun track for a long-duration test. Azimuth angle is the compass bearing.

C. Rotation Estimation

Our experiments differ in their use of the available instruments, but they share a common mathematical foundation. In each case, we must find the best estimate of C_{SF} that explains a set of observation vectors, \mathbf{u}_{S_i} , and predictions, \mathbf{u}_{F_i} . This can be written as a classic Wahba problem [18]. We wish to find the rotation matrix, C_{SF} , that minimizes the scalar-weighted cost function

$$J(\mathbf{C}_{SF}) = \frac{1}{2} \sum_{i=1}^{m} a_i \|\mathbf{u}_{S_i} - \mathbf{C}_{SF} \mathbf{u}_{F_i}\|^2,$$
(14)

where a_i is a scalar weight, m is the number of measurements, and $\|\cdot\|$ is the Euclidean norm. We solve this problem using Davenport's *q*-method [19]. Briefly summarizing this approach, we first concatenate the two sets of vectors into two matrices:

$$\mathbf{W} = \begin{bmatrix} \sqrt{a_1} \mathbf{u}_{S_1} & \sqrt{a_2} \mathbf{u}_{S_2} & \cdots & \sqrt{a_m} \mathbf{u}_{S_m} \end{bmatrix}$$
(15)

$$\mathbf{V} = \begin{bmatrix} \sqrt{a_1} \mathbf{u}_{F_1} & \sqrt{a_2} \mathbf{u}_{F_2} & \cdots & \sqrt{a_m} \mathbf{u}_{F_m} \end{bmatrix}$$
(16)

Then we calculate

$$\mathbf{B} = \mathbf{W}\mathbf{V}^T, \tag{17}$$

$$\mathbf{Q} = \mathbf{B} + \mathbf{B}^T. \tag{18}$$

We extract the cyclic components of B:

$$\mathbf{Z} = \begin{bmatrix} B_{23} - B_{32} & B_{31} - B_{13} & B_{12} - B_{21} \end{bmatrix},$$
(19)

where B_{ij} is the element of B in row i and column j. We also extract the trace of B:

$$\sigma = \text{trace}\left(\mathbf{B}\right) \tag{20}$$

Finally, we form the 4×4 matrix, **K**:

$$\mathbf{K} = \begin{bmatrix} \mathbf{Q} - \mathbf{1}\sigma & \mathbf{Z}^T \\ \mathbf{Z} & \sigma \end{bmatrix},\tag{21}$$

where 1 is a 3×3 identity matrix. The eigenvector corresponding to the largest eigenvalue of K is q_{SF} , the 4×1 unit quaternion representing the best fit rotation from F to S. This quaternion is composed of a vector component, q_v , and a scalar, q_s :

$$\mathbf{q}_{SF} = \begin{bmatrix} \mathbf{q}_v \\ q_s \end{bmatrix}$$
(22)

The desired rotation matrix, C_{SF} , can be found from the quaternion

$$\mathbf{C}_{SF} = \left(q_s^2 - \mathbf{q}_v^T \mathbf{q}_v\right) \mathbf{1} + 2\mathbf{q}_v \mathbf{q}_v^T - 2q_s \mathbf{q}_v^{\times},\tag{23}$$

where the \mathbf{q}_v^{\times} term is the skew-symmetric matrix formed from the components of \mathbf{q}_v :

$$\mathbf{q}_{v}^{\times} = \begin{bmatrix} 0 & -q_{v_{3}} & q_{v_{2}} \\ q_{v_{3}} & 0 & -q_{v_{1}} \\ -q_{v_{2}} & q_{v_{1}} & 0 \end{bmatrix}$$
(24)

At least two observations are required, but this method can be used with as many measurements as are available. In the context of this study, multiple sun vectors are helpful since the moving sun generates distinct, separate vectors. Multiple inclinometer readings do not add much new information since the gravity vector does not change. Instead, where multiple inclinometer readings are available, they are averaged to reduce noise before forming **W** and **V**.

The *q*-method is simple and robust, especially when the processing is performed offline. For online implementations QUEST [5] or FOAM [20] may provide some computational advantages, but they were not considered in this study. Switching to another algorithm would not likely improve our estimation accuracy because the largest sources of error are the raw sun sensor and inclinometer data.

D. Experiment Description

One objective of this study is to compare the effectiveness of different sensor configurations on the measurement of heading. We carried out two distinct experiments:

- 1) Heading from Sun, Gravity, and Position
- 2) Heading from Sun and Position

Each experiment has different operational implications, in addition to a slightly different mathematical formulation. We examine each concept in turn.

1) Experiment 1: Heading from Sun, Gravity, and Position: This experiment relies on measurements from the inclinometers and sun sensor as well as knowledge of the rover position on the surface of the Earth. In our experiments, position is measured using the GPS, but even coarse localization is sufficient for a reasonable heading

fix. Given this formulation, (15) and (16) become

$$\mathbf{W} = \begin{bmatrix} \mathbf{s}_S & \mathbf{g}_S \end{bmatrix}, \tag{25}$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{s}_F & \mathbf{g}_F \end{bmatrix}, \tag{26}$$

where we have used $(\forall i) a_i = 1^1$. The best fit rotation, \mathbf{C}_{SF} , can be calculated from only a single pair of observations, but in practice, \mathbf{g}_G is averaged over a short period (typically around 10 s), and converted to \mathbf{g}_S using the calibration matrix, \mathbf{C}_{SG} . Multiple sun sensor readings can be averaged if taken over a short enough time, or simply appended to W (corresponding changes must be made to V).

The best fit rotation relates frames F and S, but the heading angle α is encountered in the transformations between T and S. To extract α from the C_{SF} matrix we first calculate C_{FT} from the known position using (4). Thus we can write

$$\mathbf{C}_{TS} = \mathbf{C}_{TF} \mathbf{C}_{FS} = \mathbf{C}_{FT}^T \mathbf{C}_{SF}^T.$$
(27)

We know this equation will have the form given by (5). Multiplying out the component rotations gives a matrix of the form

$$\mathbf{C}_{TS} = \begin{bmatrix} c_{\alpha}c_{\beta} & c_{\alpha}s_{\beta}s_{\gamma} - s_{\alpha}c_{\gamma} & c_{\alpha}s_{\beta}c_{\gamma} + s_{\alpha}c_{\gamma} \\ s_{\alpha}c_{\beta} & -s_{\alpha}s_{\beta}s_{\gamma} + c_{\alpha}c_{\gamma} & -s_{\alpha}s_{\beta}c_{\gamma} - c_{\alpha}s_{\gamma} \\ -s_{\beta} & c_{\beta}s_{\gamma} & c_{\beta}c_{\gamma} \end{bmatrix}.$$
(28)

Without loss of generality, we constrain β to the range $-\pi/2 \leq \beta \leq \pi/2$, and solve for α , the desired rover heading, from the components of \mathbf{C}_{TS} :

$$\beta = \operatorname{asin}\left(-C_{TS_{31}}\right) \tag{29}$$

$$\alpha = \operatorname{atan2} \left(C_{TS_{21}}, C_{TS_{11}} \right) \tag{30}$$

This experiment is convenient since it allows for rapid measurement of rover heading, with little impact on the time needed for an extended traverse.

2) Experiment 2: Heading from Sun and Position: Although heading can be obtained very rapidly using sun and gravity measurements, there are shortcomings to this approach. Inclinometers, especially inexpensive models, are prone to drift caused by temperature fluctuations and other factors. Figure 4 shows the history of the roll-axis, γ_G , over the course of a static nine-hour test. The low frequency change parallels the sun elevation from Figure 3, suggesting a possible temperature effect.

To compensate for this effect we can reformulate the heading determination problem. Since the sun is not stationary, observations made over time will provide the necessary vectors to solve for orientation. In this case, (15) and (16) become

$$\mathbf{W} = \begin{bmatrix} \mathbf{s}_{S_1} & \mathbf{s}_{S_2} & \cdots & \mathbf{s}_{S_m} \end{bmatrix}, \tag{31}$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{s}_{F_1} & \mathbf{s}_{F_2} & \cdots & \mathbf{s}_{F_m} \end{bmatrix}.$$
(32)

¹We will continue with unit weights for the rest of the paper.



Fig. 5. Roll angle drift during extended testing.

Heading is obtained using (27)-(30), as in the previous case. Sun motion is relatively slow, so we must consider both the number of observations and the total measurement time required for a good heading fix.

E. Self-Calibration

Although the sun sensor and inclinometer are both mounted to a common rigid plate, their body frames of reference may be different. Machining tolerances on the sensor head, or even clearance in a mounting hole can create a noticeable difference between the two frames. In this section we provide a simple, and practical means of measuring C_{SG} . This technique relies only on the sensors already integrated with the rover.

To proceed, we collect a series of n moderate-length sets of sun vector readings. Between datasets, we reorient the rover in pitch and roll, ensuring that the sun remains in view. We do not need an external measure of β and γ . Using the estimation techniques of Experiment 2 we calculate \mathbf{C}_{SF_j} for each dataset. Averaging the inclinometer readings gives us \mathbf{g}_{G_j} .

For each dataset, we use the known rover position to calculate g_F from (7), and transform these vectors into S:

$$\mathbf{g}_{S_i} = \mathbf{C}_{SF_i} \mathbf{g}_F \tag{33}$$

The solution for the calibration matrix C_{SG} follows from another application of the q-method using

$$\mathbf{W} = \begin{bmatrix} \mathbf{g}_{S_1} & \mathbf{g}_{S_2} & \cdots & \mathbf{g}_{S_n} \end{bmatrix}, \tag{34}$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{g}_{G_1} & \mathbf{g}_{G_2} & \cdots & \mathbf{g}_{G_n} \end{bmatrix}.$$
(35)

The lack of reliance on external measurements makes this approach attractive, but this method is still limited by inclinometer drift. This method has performed adequately in our initial studies, but we are investigating strategies for improving the accuracy of this calibration.

IV. OPERATIONAL CONSIDERATIONS

Many factors influence the choice of landing site for planetary missions. Examples of relevent considerations include: the location targets of interest, sun availability, the presence of navigable terrain, and even the capabilities of the landing system. This section considers how the choice of landing site affects the utility and accuracy of our heading estimation algorithms. Two specific error mechanisms are of interest. First, the heading error in Experiments 1 and 2 is affected by both the rover position and its position uncertainty. Second, midday navigation at tropical latitudes can create singular orientations of the sun and gravity vectors.

In what follows, heading error, $\delta \alpha$, is computed as

$$\delta \alpha = \alpha - \alpha_{\rm true},$$
(36)

where α is our estimate of rover heading and α_{true} is the true heading. The standard deviation of heading error, σ_{α} , is also a convenient measure of the overall system performance.

A. Location Error Sensitivity

The sensitivity of the heading estimate to position error follows from the heading determination algorithms. It can be shown that a perturbation in a rotation matrix, **C**, can be written as

$$\mathbf{C} = \delta \mathbf{C} \, \bar{\mathbf{C}} \quad , \tag{37}$$

with

$$\delta \mathbf{C} \approx \left(\mathbf{1} - (\mathbf{S}\boldsymbol{\theta})^{\times} \right) \quad , \tag{38}$$

where $\bar{\mathbf{C}}$ is the operating point and $\boldsymbol{\theta}$ is the perturbation. **S** is the standard matrix that relates angular velocity to Euler angle rates or, equivalently, infinitesimal rotation vectors to Euler angle perturbations [21] (p.27). To determine the sensitivity of the heading estimate, we perturb (27), using $\bar{\mathbf{C}}_{SF}$ and $\bar{\mathbf{C}}_{FT}$ computed from the true attitude and position:

$$\delta \mathbf{C}_{TS} \bar{\mathbf{C}}_{TS} = \left(\delta \mathbf{C}_{FT} \bar{\mathbf{C}}_{FT} \right)^T \bar{\mathbf{C}}_{SF}^T$$
(39)

$$(1 - (\bar{\mathbf{S}}_{TS}\boldsymbol{\theta}_{TS})^{\times})\bar{\mathbf{C}}_{TS} \approx \left((1 - (\bar{\mathbf{S}}_{FT}\boldsymbol{\theta}_{FT})^{\times})\bar{\mathbf{C}}_{FT}\right)^{T}\bar{\mathbf{C}}_{SF}^{T}$$
(40)

In (48), θ_{FT} is the perturbation in the position and θ_{TS} is the resulting perturbation in the topocentric attitude estimate. Subtracting off the nominal value from both sides and performing some algebraic manipulation gives

$$\boldsymbol{\theta}_{TS} = -\bar{\mathbf{S}}_{TS}^{-1} \bar{\mathbf{C}}_{FT}^{T} \bar{\mathbf{S}}_{FT} \boldsymbol{\theta}_{FT} \quad . \tag{41}$$

The $\bar{\mathbf{S}}_{TS}$ matrix shares singularities with the corresponding Euler sequence (5). In this case, the equation is singular when the pitch, β , of the sun sensor in the topocentric frame is $\pm 90^{\circ}$. The form of $\bar{\mathbf{S}}_{TS}^{-1}$ is

$$\bar{\mathbf{S}}_{TS}^{-1} = \begin{bmatrix} \frac{c_{\alpha}}{c_{\beta}} & \frac{s_{\alpha}}{c_{\beta}} & 0\\ -s_{\alpha} & c_{\alpha} & 0\\ \frac{c_{\alpha}s_{\beta}}{c_{\beta}} & \frac{s_{\alpha}s_{\beta}}{c_{\beta}} & 1 \end{bmatrix} , \qquad (42)$$



Fig. 6. Analytical results showing the effect of positional uncertainty on the heading estimate. Heading error is linearly related to the error in longitude knowledge. (a) plots this line at several latitudes and (b) plots the evolution of $\delta \alpha / \delta \lambda$ as latitude increases.

and, multiplying through, the third row of (49) is

$$\delta \alpha = \frac{1}{c_{\beta}} \begin{bmatrix} c_{\alpha} s_{\beta} & -s_{\alpha} s_{\beta} c_{\phi} - s_{\phi} c_{\beta} \end{bmatrix} \begin{bmatrix} \delta \phi \\ \delta \lambda \end{bmatrix} .$$
(43)

where $\delta\phi$ and $\delta\lambda$ are the perturbations in latitude and longitude respectively, and $\delta\alpha$ is the resulting heading perturbation. For the purposes of analysis, we will set $\beta = 0$. In this case, (51) becomes

$$\delta \alpha = \begin{bmatrix} 0 & -s_{\phi} \end{bmatrix} \begin{bmatrix} \delta \phi \\ \delta \lambda \end{bmatrix} . \tag{44}$$

Letting $E(\cdot)$ denote the usual expectation operation, gives $E(\delta \alpha) = 0$. Hence, the variance of our heading error, σ_{α} , can be computed according to

$$E(\delta\alpha^{2}) = \begin{bmatrix} 0 & -s_{\phi} \end{bmatrix} E\left(\begin{bmatrix} \delta\phi^{2} & \delta\phi\delta\lambda \\ \delta\lambda\delta\phi & \delta\lambda^{2} \end{bmatrix} \right) \begin{bmatrix} 0 \\ -s_{\phi} \end{bmatrix}$$
(45)
$$\sigma_{\alpha}^{2} = s_{\phi}^{2} E(\delta\lambda^{2})$$
(46)

$$= s_{\phi}^2 \sigma_{\lambda}^2 . \tag{47}$$

Figure 5 shows the growth of heading error as the longitude error increases. In these plots, the longitude error, $\delta\lambda$, has been converted into kilometers so a direct comparison can be made. This result clearly shows that, for a level platform, errors in the heading estimate are on the same order as the longitude errors, and that these errors reduce in significance near the equator. This means that even moderate errors in positional knowledge will result in relatively small errors in heading estimate.

B. Effects of Sun Elevation / Sun Sensor Accuracy

Planetary rover missions often choose low-latitude landing sites in order to maximize available solar power. Low latitude sites see the sun at high elevations for long periods throughout each day, maximizing the effectiveness

of body-mounted photovoltaic panels. Although beneficial from an electrical standpoint, our heading solution (30) becomes singular when the sun is directly overhead (i.e., colinear with the g). More generally, the accuracy of the heading fix will degrade in the period around local noon.

Analytical treatments of the of best-fit attitude errors exist (e.g., [22] or [5]), but generally assume the presence of small errors. Although the measurement errors are small in our scenario, near singularity there is no guarantee that *attitude* errors will remain so. Pragmatically, we estimate the sensitivity of our heading determination algorithms to sun elevation using a Monte Carlo simulation. This simulation depends upon finding the dependence between heading error, $\delta \alpha$, and the sun elevation, ε . We treat the best-fit quaternion estimate of C_{TS} as the composition of the true rotation \bar{C}_{TS} and a measurement error δC_{TS} ,

$$\mathbf{C}_{TS} = \delta \mathbf{C}_{TS} \, \bar{\mathbf{C}}_{TS} \quad . \tag{48}$$

We define a temporary topocentric reference frame H such that the z-component is up and the x-component points toward the horizon below the sun. Thus, the true vectors are:

$$\bar{\mathbf{g}}_{H} = \begin{bmatrix} 0\\ 0\\ -1 \end{bmatrix} \quad \text{and} \quad \bar{\mathbf{s}}_{H} = \begin{bmatrix} \cos\varepsilon\\ 0\\ \sin\varepsilon \end{bmatrix}.$$
(49)

We apply small, perpendicular displacements to these two vectors to represent the effects of angular measurement error. The vectors are also renormalized. For simplicity, we assume that both have identical error characteristics. The errors are normally distributed in magnitude, with zero mean and standard deviation σ . The distribution is uniform in direction. The 'corrupted' vectors are denoted \mathbf{g}_H and \mathbf{s}_H . The best fit rotation between $(\bar{\mathbf{g}}_H, \bar{\mathbf{s}}_H)$ and $(\mathbf{g}_H, \mathbf{s}_H)$ is just $\delta \mathbf{C}_{TS}$; consequently, the heading, α , obtained from (30) is equivalent to the heading error, $\delta \alpha$. Repeating this process numerous times allows us to estimate σ_{α} , the standard deviation of $\delta \alpha$.

Figure 6 shows the relationship between σ and σ_{α} . At low elevations, ε , the heading error is on the order of the sensor error and varies only slightly with changing elevation. As the elevation approaches 90 degrees, σ_{α} grows dramatically. Smaller values of the sensor error σ , allows the heading algorithm to operate closer to the singularity, but performance eventually degrades. Rearranging the data gives a different perspective on the operational limitations caused by singular sun elevations (Figure 7). Given a specified *maximum* permitted σ_{α} , the curves on this plot indicate the limiting sun elevation ε_{max} as a function of sensor accuracy. When the sun is higher in the sky, heading error will not meet the requirements. Not unexpectedly, improving sensor accuracy or relaxing the heading requirements will reduce the time for which the heading fix is unacceptable.

Translating the limiting sun elevations into time constraints depends on the seasonal tilt, latitude, and target planet. For a system similar to the experiment described in this paper ($\sigma = 0.1^{\circ}$), operating on Earth, the heading error standard deviation, σ_{α} , will exceed 1° for a maximum of 46 minutes each day. The results for Mars will be similar. While not insignificant, this limitation is unlikely to hamper the use of sun-based heading determination; deadreckoning estimates of heading (e.g., gyroscope or visual-odometry) can probably be used to bridge the singularity



Fig. 7. Effect of sun elevation angle, ε , on heading error standard deviation, σ_{α} .



Fig. 8. Effect of maximum allowable heading error standard deviation, σ , on limiting sun elevation, ε_{max} .

gap. Moreover, careful scheduling of scientific observations and other operational activities can be used to sidestep the navigation restrictions altogether.

V. FIELD TESTING

To validate our sun sensing algorithms, we conducted a set of field trials at two distinct sites with a sun sensor originally designed for attitude determination on microsatellites. Below we describe the experimental setup, our datasets, and our means of determining ground truth heading in the field.

A. Test Locations

The datasets used in the experiments described in this paper were collected at two locations: the Haughton-Mars Project Research Station (HMPRS) (75°22′ N latitude and 89°41′ W longitude) and the University of Toronto Institute for Aerospace Studies (UTIAS) (43°47′ N latitude and 79°28′ W longitude). The HMPRS is situated just outside the northwest area of the Haughton impact crater, on Devon Island, in the Canadian High Arctic [23]. Haughton presents unique qualities for planetary analog studies because it offers an unusually wide variety of geological features and microbiological attributes of strong planetary analog value or potential. Haughton is also in a polar desert environment, which presents real challenges to field exploration that are analogous in fundamental ways to those expected in planetary exploration. It is worth pointing out that our Devon tests were conducted in July 2008, at which time the sun remained above the horizon 24 hours a day. Also, due to the high latitude of this site, it provides a good proving ground for sun sensor localization methods intended for use at, for example, the lunar poles. This site has been used for rover testing in the past [24–27].

B. Rover Platform

The data for the experiments in this paper were collected by a pushcart rover platform developed at the University of Toronto Institute for Aerospace Studies and shown in Figure 8. The platform includes rover engineering sensors (i.e., stereo camera, inclinometers, sun sensor, wheel odometers), a ground-penetrating radar, an on-board computer, and two independent GPS systems (one Real-Time Kinematic) used for ground-truth positioning. Although this was not an actuated rover, our focus with this platform is on problems of estimation (e.g., [28]), and thus it was entirely sufficient as a means to gather data. The sun sensor was a Sinclair Interplanetary SS-411 and is described in more detail below. The inclinometer was a Honeywell HMR-3000. The stereo camera was a Point Gray Research Bumblebee XB3 with a 24 cm baseline and 70° field of view, mounted approximately 1 m above the surface pointing downward by approximately 20° .

C. The SS-411 Sun Sensor

The sun sensor used in this study is a Sinclair Interplanetary SS-411 digital sun sensor (Figure 10). It is a very small ($\sim 30 \text{ g}$), low-power device. It boasts a 0.1° ($1-\sigma$) accuracy over a 70° half-angle field of view. An integrated microcontroller processes the readings from a linear pixel array and outputs the floating-point sun-vector in sensor coordinates. In previous studies [29], we detail the development of the sun estimation routines used in the embedded processor.

The algorithms on this device assess the quality of the detector image for each sensor measurement. Poor quality output will cause the sensor to flag the sun vector estimates as unreliable, or invalid. This behaviour appears in the collected datasets as intermittent or extended gaps in the sensor data. Clouds were the most common cause of this problem. More problematic were readings that showed strong cloud reflections close to the sun itself. These situations introduce substantial biases into the measurements.



Fig. 9. The rover platform (shown here on Devon Island).



Fig. 10. The SS-411 Digital Sun Sensor (courtesy Sinclair Interplanetary)

D. Short Datasets

For the evaluation of Experiment 1, short datasets were collected at 23 locations on Devon Island. At each location, readings from the sun sensor, inclinometer, and GPS were logged for approximately 10 seconds. As this site is near the magnetic north pole, we were unable to use a magnetic compass to obtain ground-truth heading. Instead, we devised a method of measuring the heading of the sensor using GPS. We deployed a target approximately 100 m from the sensor head, surveyed the position of the target and imaged it with the stereo camera. Converting the GPS locations to Universal Transverse Mercator (UTM) coordinates, we define $\mathbf{x}_s = [x_s \ y_s]^T$ to be the measured position of the target. The x and y coordinates correspond to Easting and Northing, respectively. The surveyed points are used to calculate θ , a known bearing in the topocentric frame:

$$\theta = \operatorname{atan2}(y_t - y_s, x_t - x_s) \tag{50}$$

Choosing the location of the target in one of the stereo images determines the bearing of the target with respect to the camera. Finally, the known transformation between the stereo camera and the sun sensor, C_{SC} determines the heading of the sun sensor, α_{true} . We currently obtain C_{SC} from the basic geometry of the sensor head, but this does not account for small misalignments (e.g., within the sensor housings). As part of our future work, we would like to develop a self-calibration routine to obtain this rotational transformation more accurately.

As we are using θ as ground-truth, it is useful to examine the accuracy this method can hope to achieve. For a generic position measurement, \mathbf{x}_i , we model the measurement noise, $\delta \mathbf{x}_i$, as drawn from a zero-mean Gaussian density with covariance Σ_i :

$$\delta \mathbf{x}_{i} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{i}\right) \tag{51}$$

The components of $\delta \mathbf{x}_i$ are defined to be

$$\delta \mathbf{x}_i = \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix},\tag{52}$$

and the components of Σ_i are

$$\boldsymbol{\Sigma}_{i} = \begin{bmatrix} \sigma_{xx,i}^{2} & \sigma_{xy,i}^{2} \\ \sigma_{xy,i}^{2} & \sigma_{yy,i}^{2} \end{bmatrix}.$$
(53)

The value of Σ_i is determined by the properties of our GPS unit. The noise model allows us to define

$$\mathbf{x}_i = \bar{\mathbf{x}}_i + \delta \mathbf{x}_i,\tag{54}$$

where $\bar{\mathbf{x}}_i = [\bar{x}_i \ \bar{y}_i]^T$ is the true position of the object.

To evaluate the accuracy of θ , we rewrite (58)

$$(x_t - x_s)\sin\theta = (y_t - y_s)\cos\theta,$$

which we expand to

$$(\bar{x}_t + \delta x_t - \bar{x}_s - \delta x_s)\sin(\bar{\theta} + \delta\theta) = (\bar{y}_t + \delta y_t - \bar{y}_s - \delta y_s)\cos(\bar{\theta} + \delta\theta).$$
(55)

Assuming $\delta\theta$ is small, we may approximate the perturbation by a first-order Taylor expansion:

$$(\bar{x}_t + \delta x_t - \bar{x}_s - \delta x_s) \left(\sin \bar{\theta} + \cos \bar{\theta} \,\delta \theta \right) \approx (\bar{y}_t + \delta y_t - \bar{y}_s - \delta y_s) \left(\cos \bar{\theta} - \sin \bar{\theta} \,\delta \theta \right) \tag{56}$$

In the absence of noise, the true heading, $\bar{\theta}$, satisfies

$$(\bar{x}_t - \bar{x}_s)\sin\bar{\theta} = (\bar{y}_t - \bar{y}_s)\cos\bar{\theta}.$$
(57)

Subtracting the operating point of the Taylor expansion, (65), from (64), and assuming that products of small terms are zero, we arrive at an approximate expression for the perturbation,

$$(\delta x_t - \delta x_s)\sin\bar{\theta} + (\bar{x}_t - \bar{x}_s)\cos\bar{\theta}\delta\theta = (\delta y_t - \delta y_s)\cos\bar{\theta} - (\bar{y}_t - \bar{y}_s)\sin\bar{\theta}\delta\theta,$$
(58)

which we solve for $\delta\theta$:

$$\delta\theta = \frac{(\delta y_t - \delta y_s)\cos\bar{\theta} - (\delta x_t - \delta x_s)\sin\bar{\theta}}{(\bar{x}_t - \bar{x}_s)\cos\bar{\theta} + (\bar{y}_t - \bar{y}_s)\sin\bar{\theta}}$$
(59)

The denominator of $\delta\theta$ is \bar{d} , the distance between the two points, such that we may write

$$\delta\theta = \frac{(\delta y_t - \delta y_s)\cos\bar{\theta} - (\delta x_t - \delta x_s)\sin\bar{\theta}}{\bar{d}} \quad . \tag{60}$$

From this expression, we can examine the resulting properties of $\delta\theta$. Its mean is

$$E(\delta\theta) = E\left(\frac{(\delta y_t - \delta y_s)\cos\theta - (\delta x_t - \delta x_s)\sin\theta}{\bar{d}}\right)$$
$$= \frac{E(\delta y_t - \delta y_s)\cos\bar{\theta} - E(\delta x_t - \delta x_s)\sin\bar{\theta}}{\bar{d}}$$
$$= 0.$$
(61)

Assuming that individual measurements are statistically independent (i.e., $E(\delta \mathbf{x}_s \delta \mathbf{x}_t^T) = \mathbf{0}$), we solve for the variance, $\sigma_{\theta}^2 = E(\delta \theta^2)$:

$$\sigma_{\theta}^2 = \frac{v_{xx} + v_{yy} + v_{xy}}{\bar{d}^2},\tag{62}$$

where we define the terms in the numerator as

$$v_{xx} = (\sigma_{yy,t}^2 + \sigma_{yy,s}^2) \cos^2 \bar{\theta},$$

$$v_{yy} = (\sigma_{yy,t}^2 + \sigma_{yy,s}^2) \sin^2 \bar{\theta},$$

$$v_{xy} = (\sigma_{xy,t}^2 + \sigma_{xy,s}^2) \sin 2\bar{\theta}.$$
(63)

We assume that our GPS units are subject to isotropic noise:

$$\Sigma_t = \sigma_t^2 \mathbf{1}, \tag{64}$$

$$\Sigma_s = \sigma_s^2 \mathbf{1}, \tag{65}$$

where 1 is the identity matrix. Under this assumption, σ_{θ}^2 simplifies to

$$\sigma_{\theta}^2 = \frac{\sigma_t^2 + \sigma_s^2}{\bar{d}^2} \quad . \tag{66}$$

Thus, knowing the properties of our GPS units (σ_t and σ_s) allows us to estimate the accuracy of our ground-truth heading using (74). We used a handheld GPS unit to survey the target, \mathbf{x}_t , and a real-time kinematic GPS to survey the rover's position, \mathbf{x}_s . Devon Island is at a high latitude and the GPS satellites are low on the horizon so we estimate $\sigma_t = 2$ and $\sigma_s = 0.4$.

E. Long Datasets

Three long datasets were collected, two on Devon Island (devonStatic1, devonStatic2) and one at the University of Toronto (utias9h). For each dataset, the rover logged sun data, inclinometer, and GPS for a number of hours. For the utias9h dataset, ground-truth heading was surveyed using the method described in Section V-D. Table II summarizes these long datasets.

Dataset	Position	Ground-Truth α_{true}	Start (UTC)	Stop (UTC)	# Sun Vectors
utias9h	43.782 N, 79.466 W	Y	2008-09-10 13:11:42	2008-09-10 21:48:53	4635
devonStatic1	75.433 N, 89.864 W	Ν	2008-07-11 15:00:22	2008-07-12 01:33:12	33561
devonStatic2	75.433 N, 89.864 W	Ν	2008-07-13 13:36:18	2008-07-13 20:42:52	2138

TABLE II A summary of static datasets.



Fig. 11. Experiment 1: Heading from sun, gravity and position. A summary of the heading error on the 307 measurements from different sites on Devon Island.

VI. RESULTS

This section presents our results for Experiments 1 and 2. In these discussions, the heading error, $\delta \alpha$, is defined as in (44). Where the ground truth is available, σ_{α} is calculated using $\delta \alpha$. Otherwise, we use $\sigma_{\alpha}^2 = \operatorname{var}(\alpha^2)$.

A. Experiment 1: Heading from Sun, Gravity, and Position

We initially used the 'Short Datasets' for this experiment. The distance between the rover and target ranged from $\overline{d} = 80$ m to $\overline{d} = 148$ m resulting in a ground-truth heading accuracy of between $\sigma_{\theta} = 1.47^{\circ}$ and $\sigma_{\theta} = 0.79^{\circ}$. Between 4 and 31 sun vectors were processed at each of 23 sites resulting in 307 data points. The mean heading error is $\overline{\delta\alpha} = 0.287^{\circ}$ and the standard deviation is $\sigma_{\alpha} = 1.02^{\circ}$. A histogram of the errors is shown in Figure 11, which shows the distribution of errors is non-Gaussian. Because the heading errors were on par with the uncertainty in our ground truth measurement, a more accurate measure of ground-truth would be necessary to evaluate the method further.

We also applied our method to the utias9h dataset, which is 9 hours long and has ground-truth heading $(\sigma_{\theta} = 0.8^{\circ})$. We ran this experiment on each of the 4635 measurements from this dataset. The algorithm exhibits a mean error of $\overline{\delta \alpha} = 0.106^{\circ}$ and a standard deviation of $\sigma_{\alpha} = 0.196^{\circ}$. A histogram of the results is shown in Figure 12, which better resembles a Gaussian distribution than Figure 11.



Fig. 12. Experiment 1: Heading from sun, gravity and position. A summary of the heading error on the 4635 measurements in the utias9h dataset.

B. Experiment 2: Heading from Sun and Position

Our objectives in conducting this experiment are twofold: we wish to compare the achievable accuracy of the heading determination to the results of Experiment 1 and we also wish to evaluate the how long we must observe to attain an acceptable heading fix. We consider the three static tests (see Table II). Even though we do not have heading ground-truth data for the devonStatic1 and devonStatic2 tests, the standard deviation in the resulting estimates give useful information. Although conditions were generally clear during the utiash9h data collection, there were several periods of scattered cloud cover. We present results for the raw dataset and a post-processed version where cloudy images have been removed. This post-processing is most important during *light* cloud-cover when a sun-vector estimate is available but distorted; when cloud-cover is heavy, the sensor is unable to locate the sun at all. The cloudy images were removed using a heuristic threshold on the integrated response across the detector image — a metric suitable for on-line implementation.

To process the data, we divided the observations into non-overlapping time windows of length Δt_w . All observations from within a time window were used to estimate α . Mean error, $\overline{\delta \alpha}$, and standard deviation, σ_{α} , for the utias9h dataset are shown in Table III. Windows with an insufficient number of data points have been omitted from these calculations. The most common cause of missing data was cloud cover (see Figure 3). The mean error and standard deviation both drop as the integration time increases (it should be noted that the small number of long datasets makes our *estimates* of $\overline{\delta \alpha}$ and σ_{α} uncertain). Cloud-correction improves the results for windows of 20 minutes or shorter but becomes less important for longer observations. The irregular behaviour of the raw mean error is somewhat perplexing, but from the distribution of errors (Figure 13), the bias in heading seems to be driven by outliers, rather than a systematic error in all the estimates. The outliers correlate with sun estimates that have been corrupted by partial cloud-cover.

Although our cloud-correction heuristic was successful in removing the worst of the distorted sun images, the

	Raw Data			Cloud Corrected			
Δt_w (min.)	Batches	$\overline{\delta \alpha}$ (deg.)	$\sigma_{\alpha}~({\rm deg})$	Batches	$\overline{\delta \alpha}$ (deg.)	$\sigma_{lpha}~({ m deg})$	
5	81	-0.34	3.27	77	-0.24	2.36	
10	43	0.36	2.40	41	0.20	2.04	
15	30	0.51	2.62	28	-0.11	1.46	
20	23	0.08	1.02	21	0.13	1.03	
25	18	-0.09	0.92	17	-0.02	0.91	
30	15	0.04	0.80	14	-0.00	0.89	
60	8	0.00	0.66	8	-0.02	0.73	

 TABLE III

 EXPERIMENT 2 RESULTS FROM UTIAS9H DATASET.



Fig. 13. Experiment 2: Error distribution for utias9h dataset prior to cloud-correction. Results are shown for 10 and 15 minute windows.

partial cloud-cover observed in the final two hours of the utias9h dataset contributed disproportionately to the error statistics. For example, the 60 min. standard deviations from the first seven hours was only 0.26° . We are currently investigating more sophisticated techniques for cloud rejection.

We can apply a similar set of time windows to the analysis of the two Devon Island datasets. If we assume that the estimates remain unbiased, then performance will be governed by the standard deviation, σ_{α} . Results from the all the long tests are shown in Figure 14. All of the curves follow a similar trend. We attribute the high initial σ_{α} values in the raw utias9h dataset to the sporadic cloud cover observed during the test; the cloudcorrected data is very similar to the Devon Island results. Additionally, the slightly elevated initial values during the devonStatic2 test could be blamed on the observed haze, but the results are inconclusive. Further tests are necessary to provide additional validation, but the results are quite encouraging from an operational standpoint. On a clear day, a five minute observation will give a 1- σ fix of about 2° and at twenty minutes, the error drops to about



Fig. 14. Experiment 2: Effect of window length on heading estimates.

1°. The Experiment 1 technique appears to be a much faster method of measuring heading, but the two techniques perform similarly when Δt_w increases to about an hour.

VII. CONCLUSIONS & FUTURE WORK

This paper has presented an experimental study of sun sensing as a rover navigational aid. Our sensor suite consists of a sun sensor, inclinometer, and clock (as well as ephemeris data). We described two techniques to determine rover heading (one involving the inclinometer and the other without). Both techniques are capable of producing heading estimates accurate to within a few degrees. The main difference is that the technique that does not use an inclinometer requires a fairly long window of sun sensor data (e.g., 1 hour) to obtain this accuracy. The novel contributions of our paper are:

- 1) Extensive experimental results (i.e., our datasets were collected at two very different sites on Earth and contain thousands of sun vectors).
- 2) Consideration of the sensitivity of heading error to sun elevation and rover position.
- 3) A method to self-calibrate the relative orientation between the sun sensor and inclinometer sensors.
- A method to obtain ground-truth rover heading in the field (i.e., without a magnetic compass) and a means to predict its accuracy.
- 5) Studies of the effect of sun sensor window length on the resulting heading estimate accuracy.

Our results are also unique in that we employed an off-the-shelf space-grade satellite sensor that requires little in the way of power and data processing. However, our methodology could also be applied to a camera-based sun sensor. Heading estimates are very sensitive to errors in the sun vector measurement caused by atmospheric effects such as clouds and haze. For terrestrial applications, it may be possible to improve robustness in these challenging operating conditions by customizing the sensor and/or processing, but we have not done this to date. We are also very interested in extending this work to the determination of global rover position using a sun sensor and inclinometer. Although not reported in the paper, we have implemented a preliminary version of position determination. The results are encouraging, but not yet adequate to be used alone for global positioning (i.e., errors are as large as 200 km). Even accounting for the difference in radii of the Earth and moon, this error would be too large for practical use. To improve these results, we believe it will be necessary to use a more accurate inclinometer than the one employed in this paper to determine heading, to account for temperature effects, and to more accurately calibrate the transformations between frames in the sensor suite itself.

Finally, we hope to extend our work to allow the use of long windows of sun vectors collected from a moving rover platform. Our long-duration tests showed significant benefit may be gained by employing a large window of sun sensor observations. However, it may not be practical to wait several hours simply to determine rover heading. One promising concept is to integrate the sun sensor with our work on visual odometry [28] to estimate both relative and absolute position/orientation simultaneously.

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